REAL OPTION VALUE

CHAPTER 2 BASIC REAL OPTIONS

Bill Way CEO of SWN argues that "we have the flexibility of dealing with commodity pricing in liquids or gas…hedge or stop or switch…with 4 rigs now in West Virginia (liquids) 2 in Pennsylvania (natural gas), we can easily switch that back as relative prices change…our share prices presents another opportunity, stop drilling and repurchase when we disconnect from the stock market"[1]. Glen Warren, President of AR states "Antero is the advantageous position of having a variety of options available for asset monetizations."[2]

Deferring drilling is a decision that is similar to the optimal timing problem using an American perpetual option model, where the investment cost is assumed to be constant, or variable (an exchange option). A similar approach is using an American option model considering uncertain output prices (natural gas) and input costs (operating expenses). As indicated, liability and security real options are critical in many strategic decisions, during the 2019 "longer lower natural gas prices", low equity prices and high risky debt yields.

One of the easiest formats for viewing such real options is the perpetual American option pricing approach, where it is assumed that over time an asset value follows an assumed pattern with a constant drift (or yield) and constant expected volatility.

2.1 PERPETUAL AMERICAN CALL OPTION

Consider a company with the opportunity to invest in a certain project; the investment cost is irreversible or irrecoverable once incurred. The investment cost K is known, or

[1] Transcript of SWN Webinar, August 2019.
[2] AR announcement December 8, 2019.

deterministic, while the present value of the project gross cash flows, V, follows a geometric Brownian motion:

$$dV = \mu V dt + \sigma V dz \qquad (2.1)$$

where μ is the growth rate or drift parameter; σ the volatility and dz the increment of a Wiener process. In a risk neutral world, or in perfect hedging which earns the riskless return, $\mu = r - \delta$, where δ is the asset yield. Since the company has the right, but not the obligation, to invest in such a project, the investment opportunity can be seen as a call option. The value, F(V), of the opportunity to invest can be represented by the following differential equation:

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} + (r-\delta)V\frac{\partial F}{\partial V} - rF = 0 \qquad (2.2)$$

Equation (2.2) is an ordinary differential equation ("ODE") with the solution:

$$F(V) = AV^{\beta_1} + BV^{\beta_2} \qquad (2.3)$$

where roots β_1 and β_2 are the positive and negative solutions of the characteristic quadratic equation[3]:

$$\frac{1}{2}\sigma^2 \beta(\beta-1) + (r-\delta)\beta - r = 0 \qquad (2.4)$$

β_1 and β_2 are respectively:

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1 \qquad (2.5)$$

[3] See Appendix 4A for the basic solution of a quadratic equation.

and

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0 \qquad (2.6)$$

Equation (2.3) gives the value of the option to invest in a project with a gross present value V. As V decreases, the value of the option to invest in the project has also to decrease and therefore the value yielded by equation (2.3) has to decrease. Moreover, due to the stochastic process followed by V, when V reaches zero it will stay there forever. This boundary condition implies that B has to be zero. Therefore, the solution can be written as:

$$F(V) = AV^{\beta_1} \qquad (2.7)$$

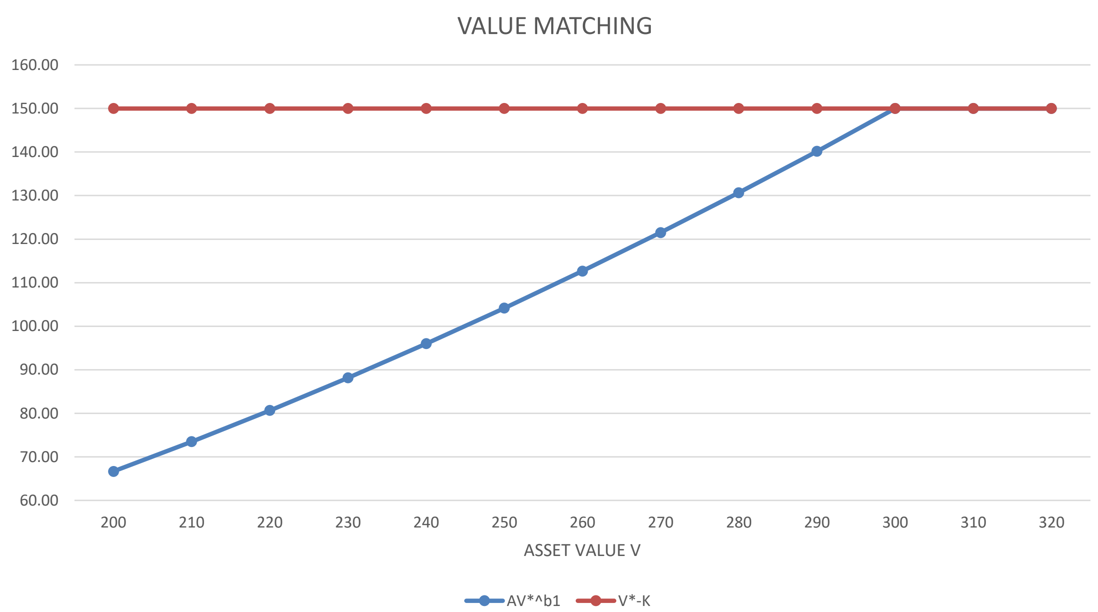
The value of the constant, A, is found subjecting equation (2.7) to two boundary conditions: **value matching** and **smooth pasting**. The value matching condition states that when V reaches a trigger or threshold value, $\hat{V}$, the option will be exercised. At that point, the investor obtains the maximum net present value of the project. At $\hat{V}$ the value of the option to invest equals the net present value of the investment.

$$A\hat{V}^{\beta_1} = \hat{V} - K \qquad (2.8)$$

The smooth pasting condition states that at the trigger value the first derivatives of the LHS and RHS of (2.8) must be equal. Notice in Figure 2.1 that the two functions in equation (2.8), the option and the net present value of the investment, meet tangentially at the trigger point.

Figure 2.1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1						Real Option Value								
2						CHAPTER 2								
3	ASSET VALUE V	200	210	220	230	240	250	260	270	280	290	**300**	310	320
4–30														
31	AV*^β1	66.67	73.50	80.67	88.17	96.00	104.17	112.67	121.50	130.67	140.17	**150.00**	150.00	150.00
32	V*-K	150	150	150	150	150	150	150	150	150	150	**150**	150	150
33	β1*AV*^(β1−1)	0.67	0.70	0.73	0.77	0.80	0.83	0.87	0.90	0.93	0.97	**1.00**	1.00	1.00
34	EXERCISE PRICE	150												
35	Yield	0.04												
36	VOLATILITY	0.20												



$$\beta_1 AV^{*\beta_1 - 1} = 1 \tag{2.9}$$

Solving (2.8) and (2.9) we obtain the constant A and the trigger function[4]:

$$A = \frac{\hat{V} - K}{\hat{V}^{\beta_1}} \tag{2.10}$$

$$\hat{V} = \frac{\beta_1}{\beta_1 - 1} K \tag{2.11}$$

Finally, substituting for A in equation (2.7), we obtain the value of the option to invest in a project where V follows a geometric Brownian motion:

[4] See Appendix 4A.

$$F(V) = (\hat{V} - K)\left(\frac{V}{\hat{V}}\right)^{\beta_1} \tag{2.12}$$

2.1.1 CONSTRÓI CASE STUDY

CONSTRÓI, S.A., a real estate developer based in the north of Portugal, can start a housing development on a farm close to Almada. The farm is derelict, and it is not possible to carry on any type of farming on the site. Immediate development of the project costs €150 million (in present value terms). Gross present value of the development is €150 million. When completed, the income from the property would be €6 million. The volatility of real estate developments has been 20%. The risk-free interest rate is 4% p.a.

CONSTRÓI's management team is worried by this uncertainty and is studying the possibility of delaying the development, since if the market falls the company will not recover the investment made. View this problem as a real call to invest in developing a site. The worried management is thinking of selling this prospect for €30 million.

What is this option worth? You do want a stake in the north of Portugal?

Figure 2.2

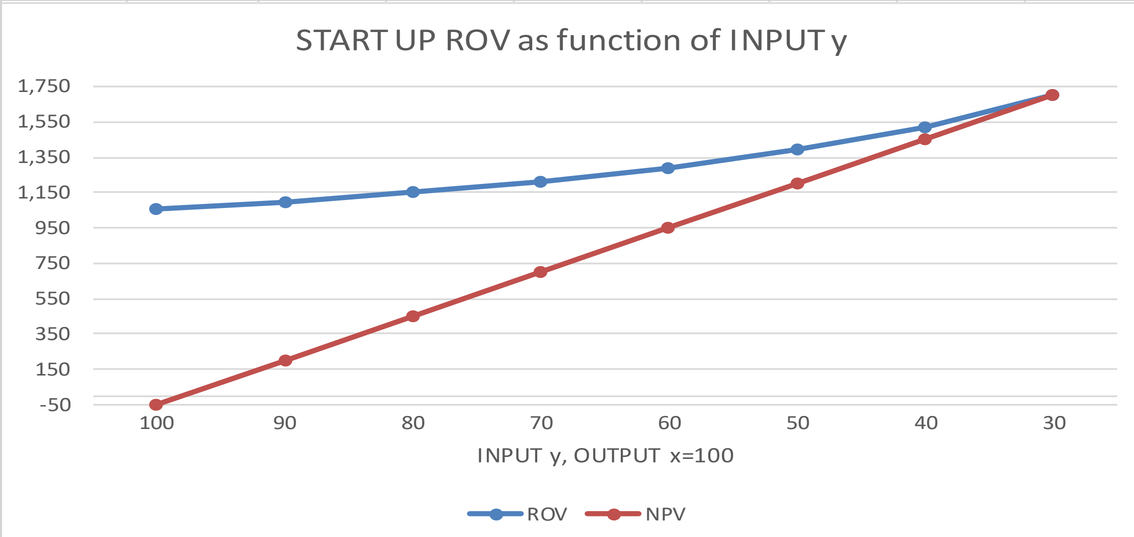
	A	B	C
1	**Perpetual American Call Option**		
2	INPUT		
3	V	150.00	
4	K	150.00	
5	σ	0.20	
6	r	0.04	
7	δ	0.04	
8	OUTPUT		
9	F(V)	37.50	IF(B3<B12,B13*(B3^B14),B10)
10	V-K	0.00	B3-B4
11	F'(V)	0.50	IF(B3<B12,B13*B14*(B3^(B14-1)),1)
12	V*	300.00	(B14/(B14-1))*B4
13	A	0.0017	(B12-B4)/(B12^B14)
14	β_1	2.00	
15	F(V)	37.50	IF(B3<B12,((B12-B4)*(B3/B12)^B14),B10)

2.2 REAL START-UP OPTION

A start-up option is similar to an investment opportunity call option, where both the unit output price and unit input cost is uncertain, possibly correlated, but the start-up cost (or switching cost) is constant.

Figure 2.3

x OUT	100	100	100	100	100	100	100	100
y IN	100	90	80	70	60	50	40	30



σx	0.40							
σy	0.30							
ρ	0.50							

Figure 2.3 shows that as the input cost declines, the ROV of this start-up opportunity increases up to the point where immediate start-up expenditures are justified, and when the real option value of the start-up option is equal to the NPV.

2.3 SHUT DOWN OPTION

A shut-down option is similar to an exit put option, where both the unit output price and unit input cost is uncertain, possibly correlated, but the shut-down cost (or switching cost) is constant.

Figure 2.4

y INPUT	200	175	150	125	100	75	50
ROV	**2780**	**2296**	**1840**	**1417**	**1029**	**681**	**381**
-NPV	**2500**	**1875**	**1250**	**625**	**0**	**0**	**0**

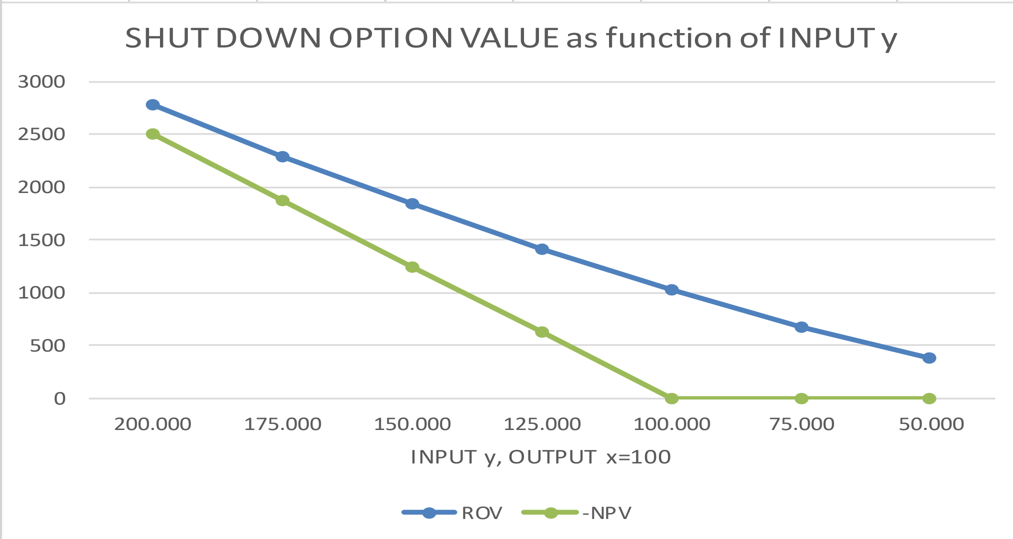


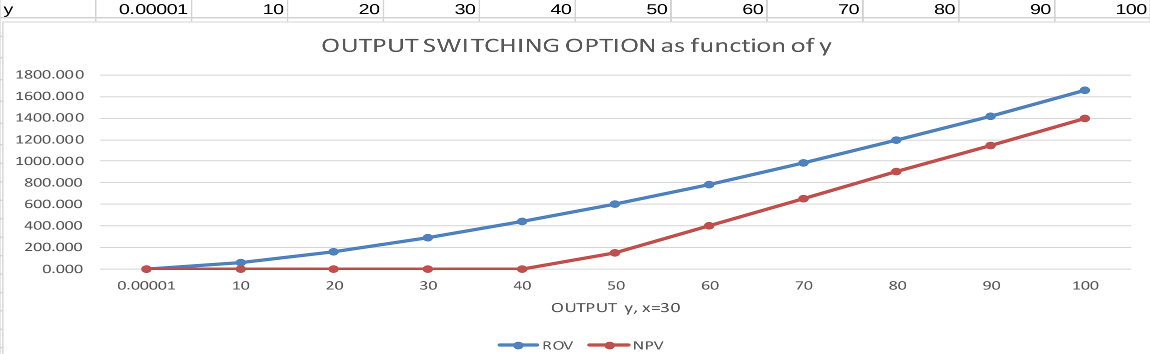
Figure 2.4 shows that as the input cost increases, the ROV and the negative NPV of this shut-down opportunity increase up to the point where immediate shut-down expenditures are justified, and when the real option value of the shut-down option is equal to the NPV.

2.4 OUTPUT SWITCHING OPTION

An output switching option is similar to an exchange option, where both of the output prices are uncertain, possibly correlated, but the switching is constant.

Figure 2.5

y	0.00001	10	20	30	40	50	60	70	80	90	100



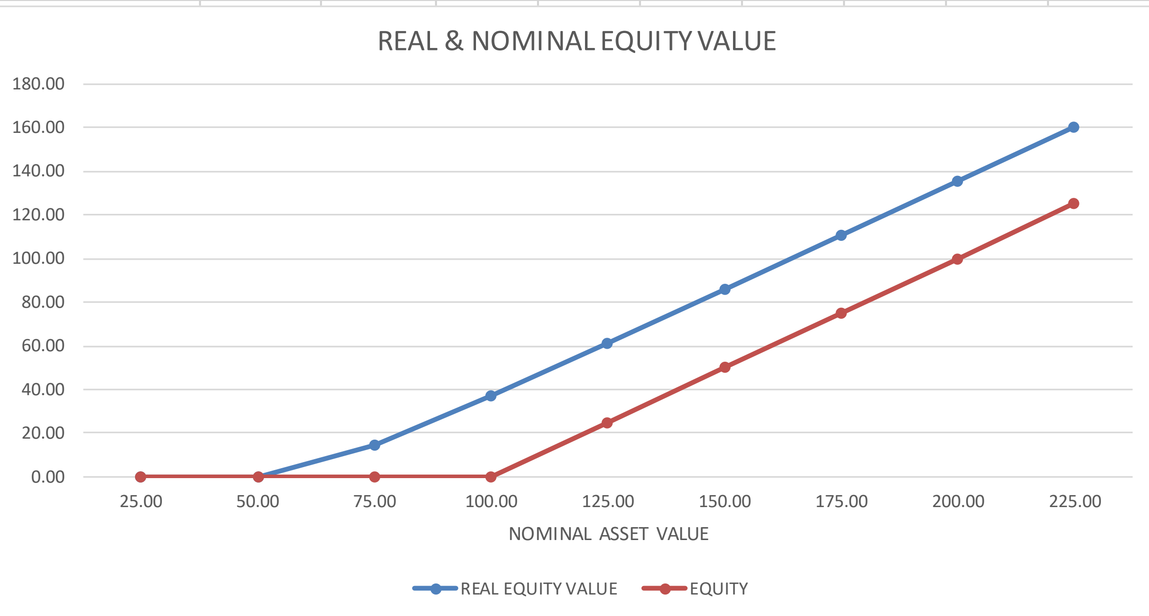
x	30
y	0
δx	0.04
δy	0.04
σx	0.4
σy	0.3
ρ	0.5

Figure 2.5 shows that as one output price y increases, while the other output price x remains constant, the ROV of this switching opportunity increases up to the point where immediate switching expenditures are justified, and when the real option value of the switch is equal to the NPV. Increasing the volatility of either output, or decreasing the correlation between the outputs, increases the real option value of the switching opportunity, as in an exchange option.

2.5 EQUITY AS REAL CALL OPTION

The real equity of a leveraged asset is equivalent to a real call option, see Merton (1973) and Leland (1994). This real option value can be compared to the accounting equity value which is equal to the book value of assets less the nominal debt. It is assumed that in default (liquidation) the value of the assets is reduced by the proportion α, and that the timing of the default is determined by the equity holder.

Figure 2.6



Coupon	6.00
δ	0.00
σ	0.20
α	0.50
NOMINAL DEBT	100.00

Figure 2.6 shows that as the nominal asset value decreases, the accounting equity =V-Debt and the real equity value both decrease to the point where immediate default is justified. Increasing the asset volatility, or reducing the foreclosure costs in default, increase the value of the asset at which default is justified, and increases the real equity value.

SUMMARY

This chapter presents five simple real option models: basic investment opportunity, start-up, shut down options, and output switching when two factors are uncertain, and the leveraged equity option with asset value uncertainty. In what circumstances can a real options valuation provide a final decision that is different from a traditional NPV approach? Real option value tends to be higher than NPV when uncertainty is high, if managers have flexibility to delay investing, switching or defaulting. If the NPV is close to zero, and there are limited immediate positive cash inflows, the time to defer a decision is valuable.

EXERCISES

EXERCISE 2.1. Townbank has the option to require Canary Wharf to buy back a perpetual lease on 100,000 square feet (SF) of space at 33 Canada Square at £42/SF. If current rental space is worth £42/δ, where the income yield δ=4%, volatility is 20% and the risk-free rate is 4%, what is the value of this option? β_1=2.

EXERCISE 2.2. To finance his education, Peter Carter wants to sell an option to Global Enterprises, whereby GE can acquire his post-AMBS services, after living expenses, forever at £75,000. Currently Peter expects to be earning £70,000 post-MoF. Nearly immortal, young Peter expects his worth to be equal to his earnings divided by 4%, his earnings volatility is 20% and the risk-free rate is 4%. β_1=2. What should he ask for this option?

EXERCISE 2.3. Citibank has an option to take up to an additional 100,000 square feet of space in Canary Wharf at R=£45/SF. Space in Canary Wharf is worth R/δ, where δ=4%, current rent is £42/SF, rent volatility is 20% and the risk-free rate is 4%, so β_1=2. What is the value of this option?

PROBLEMS

PROBLEM 2.4 An office building of 100,000 square feet in Manchester would be worth £500 per square foot, and costs £450 per square foot to build. MBA Build has received planning permission on a suitable plot of land which it wants to acquire for the development. The volatility of office buildings is 20%, interest rates 4%, and expected payout is 4%. What is the value of this land?

PROBLEM 2.5 Optimus holds a license to provide 5G in Portugal. The present value of expected net cash flows (NCF) from operations is projected to be €1,200 million, worth NCF/δ, where δ=4%. The investment cost is €1,338 million, the riskless rate is 5%, and the volatility of 5G revenues is 50%. What is the value of this opportunity, if Optimus is alone in the market?

PROBLEM 2.6. Wendy Wang is given planning permission for a modular facility suitable for either retail or residential tenants that currently costs €1,700,000 to build. The current value of perpetual rent from retail tenants is worth €1,750,000 and from residential tenants €1,600,000. Each has a volatility of 20%, an income rate of 4%, and the retail rent has a .5 correlation with the residential rent. What is this modular facility worth?